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Engineering Notes

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Asymptotic Magnetic Field Expansion in Mini-Magnetospheric Plasma Propulsion

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Introduction

MINI-MAGNETOSPHERIC plasma propulsion (M2P2) is a deep space plasma sail propulsion concept proposed by Winglee et al.[1] The concept is to generate a magnetized plasma bubble that expands into the solar wind and use solar wind pressure to accelerate a spacecraft. The magnetized plasma deflects the solar wind, and this deflection transfers momentum from the solar wind to the plasma source. The propulsive force depends directly on the cross section of the plasma bubble, which in turn depends on how the magnetic field in the plasma drops off at large radii. Winglee et al.[1] presented magnetohydrodynamic simulations in three dimensions that show that the generation of plasma changed the asymptotic magnetic field behavior at large radii from the $1/r^3$ of a dipole in vacuum to approximately $1/r$. The exact dependence is very important the for application of M2P2 to spacecraft propulsion because the ratio of the stagnation point in the solar wind to the initial plasma radius is expected to be large and a small change in the exponent would make a large difference in the thrust produced. In this Note, we show that, for an idealized M2P2 in a constant velocity, spherically expanding, perfectly conducting, high beta plasma, the magnetic field falls off as $1/r$ in agreement with numerical simulations. This slow falloff means a large interaction region between the M2P2 and the solar wind and supports the previous studies of the effectiveness of plasma sail propulsion.

Analysis

We assume a constant velocity, spherically expanding, perfectly conducting, high beta plasma. Neglecting electron pressure and electron inertia, the electric field $\boldsymbol{E}$ in the plasma can be written as

$$\boldsymbol{E} = -\boldsymbol{v} \times \boldsymbol{B} \tag{1}$$

where $\boldsymbol{v}$ is the fluid velocity and $\boldsymbol{B}$ is the magnetic field. The plasma mass continuity equations in the frame of the moving plasma is

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho \nabla \cdot \boldsymbol{v} = 0 \tag{2}$$

where ρ is the plasma mass density. To derive the magnetic field, we combine Faraday's law with Eq. (1):

$$\begin{aligned} -\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla \times \boldsymbol{E} = -\nabla \times (\boldsymbol{v} \times \boldsymbol{B}) \\ &= -[\boldsymbol{v}(\nabla \cdot \boldsymbol{B}) - \boldsymbol{B}(\nabla \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} - (\boldsymbol{v} \cdot \nabla)\boldsymbol{B}] \end{aligned} \tag{3}$$

Using the fact that the magnetic field is divergence free and combining with Eq. (2), we obtain

$$\begin{aligned} \frac{\partial \boldsymbol{B}}{\partial t} &= -\boldsymbol{B}(\nabla \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} - (\boldsymbol{v} \cdot \nabla)\boldsymbol{B} \\ \frac{\partial \boldsymbol{B}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{B} &= -\boldsymbol{B}(\nabla \cdot \boldsymbol{v}) + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} \\ \frac{\mathrm{d}\boldsymbol{B}}{\mathrm{d}t} &= \boldsymbol{B}\left(\frac{1}{\rho}\right)\frac{\mathrm{d}\rho}{\mathrm{d}t} + (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} \\ \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\boldsymbol{B}}{\rho}\right) &= \frac{1}{\rho}(\boldsymbol{B} \cdot \nabla)\boldsymbol{v} \end{aligned} \tag{4}$$

This result, Eq. (4), can be found in textbooks on plasma physics.[2]

In one-dimensional spherically symmetric flow, $\boldsymbol{v}$ is radial and $\boldsymbol{B}$ has polar and radial components. In the equatorial plane, $\boldsymbol{B}$ has only a polar component:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{B_\theta}{\rho}\right) = \frac{1}{\rho}\frac{B_\theta v_r}{r} \tag{5}$$

For a constant radial velocity, the mass density decreases with the inverse square of the radius, $\rho \propto 1/r^2$. Using the definition of the radial velocity $v_r \equiv \mathrm{d}r/\mathrm{d}t$, we find that the magnetic field strength drops inversely with radius:

$$B_\theta \propto 1/r \tag{6}$$

in agreement with the numerical results of Winglee et al., without inclusion of additional current sources or the solar wind.

In this idealized example, the magnetic field expansion is driven by the plasma, whose kinetic energy is assumed to be large enough that the expansion velocity remains constant. If plasma is continually being added and expanding, then the magnetic coil would feel a back electromotive force corresponding to the increasing inductance L. This requires additional electrical power P_{el}, to the magnet, beyond the ohmic losses considered by Winglee et al.[1]:

$$P_{\mathrm{el}} = RI^2 + I^2\frac{\mathrm{d}L}{\mathrm{d}t} \tag{7}$$

where R is the wire resistance and I is the magnetic current. The inductance is related to the magnetic field energy in the volume V of plasma by

$$\frac{1}{2}LI^2 = \int_{V_p} \frac{1}{2\mu_0} B^2\,\mathrm{d}V \tag{8}$$

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where μ_0 is the permittivity of free space. In the constant velocity approximation this magnetic energy is supplied by the Poynting flux:

$$(1/\mu_0)(\boldsymbol{E} \times \boldsymbol{B}) \cdot \boldsymbol{A} = (1/\mu_0)B^2\boldsymbol{v} \cdot \boldsymbol{A} \tag{9}$$

where $\boldsymbol{A}$ is the area across which the plasma flux is supplied.

Conclusions

The rigorous analytical derivation shows that $1/r$ scaling is consistent with magnetic field lines "frozen" in a spherically expanding plasma and not just a result of a numerical simulation. The magnetic field energy is imparted at the source and does not rely on the properties of an interacting solar wind for additional energy. However, the derivation is idealized and still leaves open critical questions concerning the application of M2P2 to real spacecraft propulsion.

References

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[2]Landau, L. D., and Lifshitz, E. M., *Electrodynamics of Continuous Media*, Pergamon, New York, 1960, p. 216.

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Algebraic Correlation for High-Speed Transition Prediction on Sphere Cones

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Introduction

THE prediction of the occurrence of boundary-layer transition remains as one of the most challenging problems in basic physics. Transition occurs when disturbances present in a laminar flow are amplified and grow to the critical amplitude necessary for breakdown to a turbulent flow. The complexity of this problem is compounded by some unusual reversals in transition behavior for hypersonic flow. A discussion of the many paradoxes associated with high-speed transition may be found in Ref. 1. At present, there are no general empirically derived correlations for predicting hypersonic transition. Limited success in correlating transition data has been achieved by restricting the empirical database and thereby limiting the generality of the correlations. However, there are points in some of these correlations where the spread in uncertainty can vary as much as an order of magnitude.[2]

Future opportunities for acquiring high-speed boundary-layer transition data through flight testing will be rare due to the exceedingly high costs involved. Thus, the current challenge is to exploit the existing experimental database and to develop a method that reliably predicts the onset and movement of boundary-layer transition. The purpose of this Note is to describe a simple algebraic correlation for predicting laminar-to-turbulent flow transition on the frustum of smooth, nonablating sphere cones in free flight. This correlation is based on an analysis of a large set of well-documented high-speed ground- and flight-test data using a transition Reynolds number with a length scale equal to $(\theta^3/S)^{1/2}$, where θ is the momentum thickness and S is the body-surface streamlength. Because most reentry configurations employ an ablative thermal protection system, the applicability of the present correlation to bodies with ablating surfaces is examined.

Analysis and Data Selection

Inconsistencies in the data analysis can lead to scatter in the correlation process. For example, different investigators will employ different methods to predict edge conditions and, hence, will compute different local edge Reynolds numbers. Thus, transition location is not a well-defined quantity when reported in terms of a transition Reynolds number. For this reason, only results from experimental investigations that report actual transition locations are utilized. The most accurate method for determining transition onset location is believed to be onboard measurement of the sudden rise in surface heat transfer rates. Consideration for data selection required this method of transition detection for reentry flight tests. For ballistics-range ground tests, however, transition locations are typically estimated from shadowgraphs during low-Mach-number runs or from drag measurements during high-Mach-number runs.

Ballistics-range data are invaluable because they are free of the acoustic disturbances that contaminate conventional shock- and wind-tunnel measurements. However, results from shadowgraphs generally yield transition locations roughly halfway between onset and end.[3] Transition onset locations can be estimated from optically determined shadowgraph results using information given in Refs. 3 and 4. A comparison of transition detection methods given in Ref. 3 shows that peak temperature recovery factor locations roughly coincide with average transition streamlengths determined from shadowgraphs, that is, $S_{\text{peak}} \approx S_{\text{optical}}$. In addition, quiet-tunnel data from Ref. 4 suggest that the value of $S_{\text{onset}}/S_{\text{peak}}$ approaches 0.8 for high edge unit Reynolds numbers. This implies that $S_{\text{onset}}/S_{\text{optical}} \approx 0.8$. Although no analogous correction exists for drag-inferred transition data, they are included here as reported in the literature to increase the high freestream Mach number statistics associated with the database.

In addition, most reentry applications employ a nosetip made from an ablating material to withstand the high temperatures generated near the stagnation point during flight. An additional complication can be avoided by restricting the data selection to geometries with one type of nosetip material. This restriction is not a severe one because the vast majority of the available flights employed some type of graphitic material. Thus, only those reentry configurations with a graphitic nosetip are included in the present empirical database.

Other requirements, which substantially reduce the size of the available database, include 1) a sphere–cone configuration having a nonablating frustum material; 2) a high-speed free-flight flow environment, where the freestream Mach number is greater than 3.5; 3) a small total angle of attack at transition, namely, less than 1.25 deg for ballistics-range data and less than one-tenth of the half-cone angle for reentry data; and 4) a reentry or descent-phase laminar-to-turbulent flow transition. The final empirical database consists of 101 points, which were selected from the ballistics-range results of Potter[5] (5 points), Reda[6] (26 points), and Sheetz[7] (21 points) and the flight-test data of Berkowitz et al.[2] (32 points), Johnson et al.[8] (8 points), and Krasnican and Rabb[9] (9 points). This database covers a wide parameter space in terms of nose radius, half-cone angle, and freestream conditions. With respect to geometric parameters, the value of nose radius varies from 0.001 to 4 in. (0.00254 to 10.16 cm), whereas the value of half-cone angle varies from 5 to 22 deg. As for freestream conditions, the Mach number varies from 3.5 to 23.1, whereas the unit Reynolds number varies from 0.7×10^6/ft to 76.7×10^6/ft (2.3×10^6/m to 251.6×10^6/m).

The goal here is to determine those parameters that accurately correlate frustum transition. This is not an easy task because there have been many previous attempts to correlate transition data. A good example of the difficulty involved may be found in Ref. 2, where

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